

Please complete this worksheet by **September 24th, 2021** by **11:59pm**.

Once you upload a picture of your work ([here](#)), the solutions will become available so you can study for the weekly quizlet, which may draw one problem from this week's worksheets.

### Problem 1

Read section 5.3 of the *Book of Proof* by Richard Hammack. Then read the following (poorly written) proof of the statement: *If  $n$  is even, then  $n^2$  is even.*

#### Proof:

1. Let  $n =$  an integer.
2. Suppose  $n$  is even.
3. Then  $n = 2k$ .
4.  $n^2 = (2k)^2$ ,  $(2k)^2 = 4k^2$ , so  $4k^2 = 2(2k^2)$
5. Since  $(2k^2)$  is an integer, I've shown it is even.

These sentences in the proof are numbered to make it easier to reference specific lines in your answer.

- (a) Identify sentences that violate Hammack's mathematical writing guidelines and explain why.  
Note: a sentence can violate multiple guidelines, and so can be included multiple times.
- (b) Rewrite the proof so that it follows Hammack's mathematical writing guidelines.

### Problem 2

Prove:

$$|x + y| \leq |x| + |y|$$

$\forall x, y \in \mathbb{R}$ , where  $|\cdot|$  represents the absolute value of whatever is in between the vertical bars.

*Hint: use a proof a by cases with the cases (1)  $x > 0, y > 0$ , (2)  $x < 0, y < 0$ , (3)  $x < 0, y > 0$  and (4)  $x = 0, y \in \mathbb{R}$ . Case (3) should be split into subcases: (3a)  $x + y = 0$ , (3b)  $x + y > 0$  and (3c)  $x + y < 0$ .*