Please complete this worksheet by **October 1st, 2021** by **11:59pm**.

Once you upload a picture of your work (here), the solutions will become available so you can study for the weekly quizlet, which may draw one problem from this week's worksheets.

Problem 1

What is wrong with the following proofs?

(a) The sum of the first n odd numbers is n^2 for $n \geq 1$.

Proof. We use a proof by induction on $n \geq 1$.
Let the induction hypothesis be $p(n) =$ *the sum of the first odd n numbers is n^2*.

Base case: For $n = 1$, the sum of the first odd number is $1 = 1^2$, so $p(1)$ is true.

Inductive step: For $n = 2$, we look at $1 + 3 = 4 = 2^2$. For $n = 3$, we look at $1 + 3 + 5 = 9 = 3^2$. Continuing in this way, we see that the sum of the first n odd numbers is n^2.

Therefore, for all $n \geq 1$, $p(n)$ is true. □

(b) The sum of the first n numbers is $(n^2 + n)/2$ for $n \geq 1$.

Proof. We use a proof by induction on $n \geq 1$.
Let the induction hypothesis be $p(n) =$ *the sum of the first n numbers is $(n^2 + n)/2$*.

Base case: For $n = 1$, the sum of the first number is $1 = (1^2 + 1)/2$, so $p(1)$ is true.

Inductive step: Assume $p(n)$ is true for $n > 1$. Then if we consider $p(n + 1)$, we have

$$1 + 2 + \cdots + n + 1 = ((n + 1)^2 + (n + 1))/2.$$

Now if we subtract $n + 1$ from both sides and manipulate, we have

$$1 + 2 + \cdots + n = (n^2 + n)/2,$$

which is $p(n)$, which we know is true.

Therefore, for all $n \geq 1$, $p(n)$ is true. □

(c) In a set of n horses ($n \geq 1$), all horses have the same color.

Proof. We use a proof by induction on the induction variable n, the number of horses in a set. Let the induction hypothesis be: $p(n) =$ *in a set of n horses, all horses have the same color.*

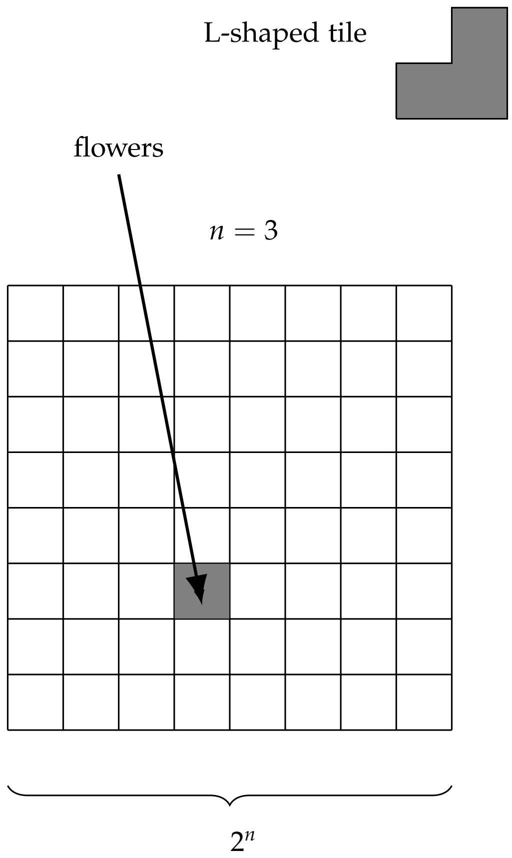
Base case: Our base case is for $n = 1$, i.e. for a single horse, which has the same color as itself.

Inductive step: Assume that $p(n)$ is true. That is, in a set of n horses, they all have the same color. Now, consider a set of $n + 1$ horses.

$$\overbrace{H_1,\ H_2,\ \ldots,\ H_n}^{\text{same color}},\ H_{n+1}$$
$$H_1,\ \underbrace{H_2,\ \ldots,\ H_n,\ H_{n+1}}_{\text{same color}}$$

Since H_1 is the same color as the horses: $H_2, \ldots, H_n$ and H_{n+1} is the same color as the horses: $H_2, \ldots, H_n$, then H_1 and H_{n+1} are the same color. By induction, this means that a set of n horses all have the same color.

□



Problem 2

The town of Middlebury has just hired you to tile one of its courtyards. The outline of the courtyard is a square with side length 2^n for $n \geq 0$, and it *must* contain space (equivalent to one tile) for a pot of flowers. For whatever reason, all the normal tiles are sold out at Agway; all they have left are L-shaped tiles. At a town hall meeting, no one can decide where to put the pot of flowers. To make things worse, people are getting angry because they think there is no way you can tile the courtyard with L-shaped tiles while still leaving space for the flowers.

Prove that you can tile any $2^n \times 2^n$ ($n \geq 0$) courtyard with L-shaped tiles, leaving one space for a pot of flowers.