

Please complete this worksheet by **October 4th, 2021** by **11:59pm**.

Once you upload a picture of your work ([here](#)), the solutions will become available so you can study for the weekly quizlet, which may draw one problem from this week's worksheets.

### Problem 1

The Fibonacci numbers  $F(0), F(1), F(2), \dots$  are defined as follows:

$$F(0) = 0$$

$$F(1) = 1$$

$$F(n) = F(n-1) + F(n-2)$$

(a) Prove using regular induction that for all  $n \geq 1$ ,

$$F(n-1) \cdot F(n+1) - F(n)^2 = (-1)^n.$$

(b) Prove using strong induction that for all  $n \geq 0$ ,

$$F(n) = \frac{\alpha^n - \beta^n}{\sqrt{5}}, \quad \text{where } \alpha = \frac{1 + \sqrt{5}}{2}, \beta = \frac{1 - \sqrt{5}}{2}.$$

*Hint: note that  $\alpha$  and  $\beta$  are the two roots of  $x^2 - x - 1 = 0$ . This means that  $\alpha + 1 = \alpha^2$  and  $\beta + 1 = \beta^2$ .*