

Please complete this worksheet by **October 6th, 2021** by **11:59pm**.

Once you upload a picture of your work ([here](#)), the solutions will become available so you can study for the weekly quizlet, which may draw one problem from this week's worksheets.

Problem 1

Consider the following matrix, whose entries are a function of θ :

$$R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}.$$

- Let \vec{v} be the vector $[-2, 2]^t$. What is $\vec{u} = R(\theta)\vec{v}$ for $\theta = 3\pi/4$? What are your ideas on what this means geometrically?
- Verify that $R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)$. *Hint: you might need to look up some trig identities.*
- Prove using induction that $R(\theta)^n = R(n\theta)$ for $n \in \mathbb{N}$. Note that $R(\theta)^n$ refers to the matrix $R(\theta)$ being raised to the n^{th} power. *Hint: in your inductive step, you can use the result of part (b) - i.e. with $\theta_1 = n\theta$ and $\theta_2 = \theta$.*
- Calculate $R(\theta)^t R(\theta)$. What are your ideas on what this means geometrically?