Please complete this worksheet by **October 15th, 2021** by **11:59pm**.

Once you upload a picture of your work (here), the solutions will become available so you can study for the weekly quizlet, which may draw one problem from this week's worksheets (there is no quizlet this week, but one of these problems may appear in the quizlet for next week).

Problem 1

(Adapted from *The Fascinating World of Graph Theory*). An art gallery contains 12 rooms r_1, r_2, ..., r_{12}, each of which contains an expensive painting. Figure 1 shows the blueprint for the gallery which shows the doors between each room.

- Represent the art gallery as a graph. What are the vertices and edges? Draw the graph.
- Do there exist **four** rooms where security guards may be placed so that every room either contains a guard or neighbors a room containing a guard? If so, in which four rooms would you place the security guards? Explain your answer.

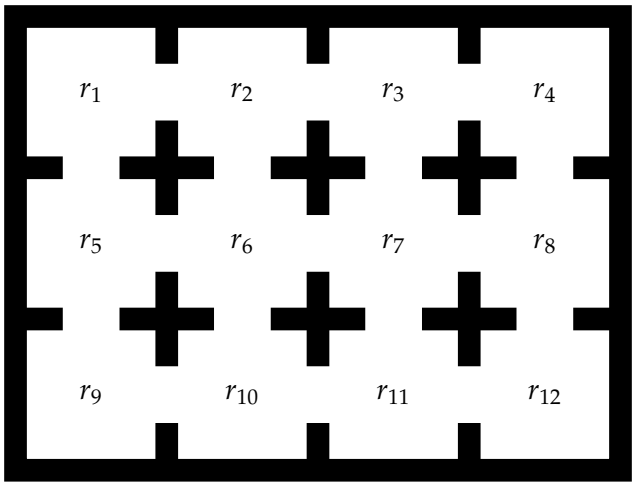

Figure 1: Art gallery layout.

Problem 2

At a party with 31 people, each person said they spoke to *exactly* 9 people. Prove (using a proof by contradiction), that this situation is impossible.